Theorizing Mathematical Unitizing through Fiber Crafts

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Abstract: This article examines how fiber crafting can develop mathematics learning and learners. Extending the constructionist paradigm with relational materialist principles, this paper advances the notion of "materialized action," which describes the natural inquiry process that results through emergent patterns between learners and the materialized traces of their actions. This paper takes a qualitative approach, combining a design and intervention phase examine fiber crafts (here knitting) and engagement in a "powerful idea" (i.e., unitizing in multiplicative proportional reasoning) as an illustration of how we can better understand micro-developmental learning processes, and advance constructionist theory.

Introduction

Identifying relatable contexts to practice challenging mathematics topics, such as multiplicative proportional reasoning, remain a challenge in mathematics learning. In this study, we build on fiber crafts a rich context of mathematics learning and examine how fiber crafting can develop mathematics learning and learners. Toward this end, we asked: How does knitting develop mathematics learning and learners and under which conditions? In the context of an out-of-school workshop developed by crafting professionals and mathematics educators, this qualitative study examines knitting as process of creating units and engagement in multiplicative proportional reasoning. This effort combines research on the use of textile crafts for learning advanced mathematics (e.g., Greenfield & Childs, 1977; Peppler et al., 2020) with a relational materialist lens on learning (Hultman & Lenz-Taguchi, 2010) to capture, analyze, and theorize how materials prompt human development and learning. As part of a longer-term qualitative study that focused on capturing evidence of learning via fiber crafts (e.g., Keune et al., 2021), this paper presents a close analysis of the micro-developmental engagement of knitting with youths to show how material changes led to engagement in powerful mathematical ideas of unitizing and proportional relationships. While the full study shows these ideas unfold in three fiber crafts (i.e., knitting, crochet, and pleating) we focus on presenting findings on knitting in this paper. Through this examination, we advance the notion of materialized action as a micro-developmental condition under which epistemic understanding emerges. At its core, materialized action can be conceptualized as the patterns of action in the construction of an artifact. Materialized action allows for the reformulation of ideas (if the physical outcome was not intended) and the co-construction between learner and materials. It has the potential to disrupt in mathematics education.

Proportional reasoning and unitizing in mathematics

This study takes as a starting point the theory of constructionism, which posits that learning occurs best when individuals design physical (or digital) constructions that can be shared, representing cognitive transformations that happen as learners actively engage with domain concepts. Working out reasons for why designs fail and adjusting designs is one important way to deepen understanding of mental models and concepts (Papert, 1980; Kafai, 2006). The types of materials used for learning are not without consequence, as materials, and the relative marginalization of other materials, shape domains in formative ways. Friedman (2018) details how compass and straight-edge produced a range of mathematical techniques and practices that subverted and marginalized other mathematical principles based on paper folding. In a recent workshop organized at the Technical University of Munich and Deutsches Museum, Friedman and Zetti (2023) trouble the immateriality of mathematics by highlighting the distinct role paper played in knowledge production within mathematics and computing contexts. This has manifold consequences for how we conceive of mathematics learning. Additionally, while math learning and fiber craft learning were historically placed in opposition to each other in European and American schools (Harris, 1997), researchers have observed ample connections between textile crafts and math that belie this separation, such as in knitting, crochet, cross-stitch, quilting, needlepoint, and tatting, among others (e.g., Belcastro & Yackel, 2011). Other work demonstrated mathematical learning through textile craft engagement, such as sewing of tents and costumes, knitting, crochet, and weaving (Peppler et al., 2022).

In the constructionist tradition, researchers look for powerful ideas that are persistently difficult as taught using traditional approaches. One such powerful idea, which we examine in this study, is unitizing in multiplicative proportional reasoning (PR). PR is the understanding of the multiplicative part-whole relations between rational quantities and is a predictor of future mathematics achievement (de la Torre et al., 2013; Boyer & Levine, 2015). Persistently, PR has been challenging to learn (Lobato & Thanheiser, 2002); often, young learners try to use additive instead of multiplicative strategies (e.g., incorrectly solving 2/3 = x/6 by adding 3 to both numerator and denominator instead of multiplying both numerator and denominator by 2; e.g., Dooren et al., 2010). Unitizing, the partitioning into composite units, is a foundational concept for multiplicative and proportional reasoning and is difficult for children to develop (Lamon, 1992). There is a powerful possibility in feeling and practicing units across multiple materialities as to "disturb narrow (and perhaps white, western, male) images of mathematics—and to open up opportunities for a more pluralist school mathematics," that draws on different cultural experiences, materialities, and abilities (de Freitas & Sinclair, 2020, p.2). Missing from this prior work is how units can be dynamic and tangibly produced (rather pre-formed) as well as how units build over time.

Methods

This qualitative study explored what features of activities could lead to exploration of proportional reasoning by analyzing data from a three-day fiber crafts camp in which youth performed the craft activities to understand how the crafts supported engagement with PR. The camp took place over the course of three consecutive days (4 hours each). Each day youth learned a new craft and created a project with the craft: 1) Crochet a bag, 2) knit a bag o, and 3) sew a pleated bag. 17 middle-school-aged youths (9-12 years old) participated in the craft camp (16 female, 1 male). This is an age at which PR is typically taught (Common Core State Standards Initiative, n.d.) as well as the age at which girls—and other underrepresented youth—begin to wind down on interest with STEM (Corbett & Hill, 2015). Two participants were joined by their parents to support language translation. For analytical purposes, we focused on three focal youth based on active participation.

The data sources included projects created by the research team. Wideo recording of the youth camp observed the youth-produced proportional reasoning across projects through material unitizing and shaping (40 hours). The video showed youths' hands and faces as they worked on their projects. Data sources also included 231 photographs of youth projects that showed detail of the projects. We first analyzed the projects by the research team through 1) verbal descriptions of the step-by-step craft process, 2) visual representations that showed the emergent material and craft patterns, and 3) corresponding mathematical notation of the RP in the crafts that represented the rules that governed the patterns. As per constructionist philosophy, the abstractions into mathematical notation were not a part of the camp. The analysis of the video recordings of the youth camp focused on how youth produce PR across fiber crafts through material unitizing (i.e., how crafters produced units in the materials). Our analysis of the photographs of youth projects closely observed stitches to reconstruct the mathematical doing that occurred to produce the project. We focused on the differences between planned and implemented projects (e.g., in relation to stitch size) as reference for mathematical processes.

Findings

In contrast to the use of established units as the basis for ratios and proportional relationships, fiber crafting begins with an initial stitch unit that users define through their choice of materials and their body's relationship to their manipulation. Crafters reason with multiplicative part-whole relations as rhythmic and repeated movements of people and materials arrange and multiply stitch units into pattern units, which are multiplied again into a project unit. In this study, we identified three levels of unitizing: *Stitch units* form the basis of proportional relationships when considering the number of stitches per row. *Pattern units* emerge by bringing stitch units in relation. We define a pattern as a form or model used for imitation. *Project units* from through the combination of completed patterns. The project unit shows the mathematical connections even more clearly.

How knitting develops mathematics learning and learners

Katie, a 10-year-old knitter, produced stitch units only to unravel them and to restart 12 times, working to establish a consistent feel for her stitch units. Where initial stitch units were loose and irregularly shaped, as Katie got into a routine, her stitch units became tighter and more uniform. Katie and a neighboring youth also explored stitch units through a conversation about arm knitting, a knitting technique that uses the arms of the crafter in place of knitting needles (see Table 1). Together, the youth determined that the stitches would be gathered on one arm and picked up by the other arm (see Table 1, panel 1 and 2). Through gestures, the youth compared the effects of using different materials (i.e., wooden needles vs. arms as needles) on one's personal stitch unit (see Table 2, panel 3 to 5). Through her body posture and arm gestures, Katie expressed how the size of a needle affected the amount of yarn needed for a stitch as well as the size of a stitch unit (see Table 1, panel 6).

Table 1
Transcript of a conversation about arm knitting that involves stitch units and pattern units.

 1) Katie: "Your arms are like needles." Katie turns to Sarah and lifts her arms. Katie's arms become needles.	 2) Sarah: "Like this?" Sarah lifts her project. Katie drops her arms and nods. Sarah introduces her project as a comparison.	 3) Katie: "I don't know how exactly." Katie lifts her arms and twists them. With arms as needles, Katie explores how arm knitting would work.	 4) Katie: "Now that I think about it, it's like the needles." Katie lifts her project and points at the needles. Katie suggests that arm-needles act similar to wooden needles.
 5) Sarah: "Ah." Katie: "Yes." Sarah lifts her left arm and grabs it with her right hand at three places. Katie nods. Both knit on. Sarah's arm becomes a needle and the grabbing motion becomes stitches on the needle.	 6) Katie: "If you used this arm, it'd be stitches that big." Katie holds her hands one foot apart. Katie shows how the size of the project becomes larger with arms as needles.	 7) Katie: "Lalala" Singing, Katie waves her arms. The waves become stitches and Katie adds a few stitches to her imaginary project.	 8) Katie: "Then you have that much." Katie holds her arm two feet apart. The imaginary project grew over twice in size and, thus, at a faster rate compared to using wooden needles.

Starting over allowed Katie to practice a sense of her personal gauge, reflective of pattern units. With an increasing number of unraveled projects, Katie considered how the number of stitches she cast on would relate to the size she produced, counting the stitches as she cast on her needle. Katie also compared knitting as getting a physical sense of the size of a stitch in relation to the created pattern unit in space, and, more specifically, the length of a row of stitches (see Table 2, panel 7 and 8). This is relevant because needle size is one aspect of how knitters conceive of their personal pattern unit, which shapes the look and size of a stitch unit (i.e., how big or how loose it is). As she worked, each stitch reconstructed the rectangular stitch unit that became the basis for a proportional relationship while this reconstruction was a part of forming the pattern unit. Each pattern of stitches thus formed another unit of the mathematical materialized action. The project unit showed the mathematical connections even more clearly. A knitted stitch unit is rectangular in shape and, thus, the stitch height is unequal to (shorter than) stitch length. This produces a proportional relationship, which in knitting looks like a performance centered on the gauge of a knit. As she worked, Katie noticed the proportional relationship at the site of the project unit, when she realized her project unit did not match the drawn pattern unit and that her stitch unit was not square.

Moving across three units provided space for iterative material exploration (i.e., through the undoing and redoing of stitch units) and drawing relationships across units, which brought about the implementation of proportional reasoning, but in greater complexity than what we would find in traditional classroom exercises. Learning about the epistemic idea is moving between units and is building toward larger constructions. This performative comparison of knitting with needles and knitting with arms was indicative of Katie's developing sense the craft material that affected the production of a stitch unit, the basic element for PR within knitting. Yet, when moving from stitch unit to pattern unit, we start to see intersections and moving back and forth across units.

Discussion and Implications

Through our analysis of knitting across units, we can see how learners engage epistemically in mathematical ideas across different levels and complexities. The personalized stitch unit becomes a materialized action that crafters can recognize with hands and eyes. Beyond building units, crafters can zoom in on the combination of units into pattern units as a way to think about what the combination of units can produce that is larger than the unit itself. Taken together, this advances a notion of materialized action, resituating the "doing of" mathematics as a natural inquiry process that results through emergent patterns between learners and the materialized traces of their actions. Types of units can be simultaneously and separately engaged. By working across units, crafters engage materialized actions that provide opportunities for proportional reasoning. Materialized actions integrate (rather than exclude) worldly concreteness into mathematics, promising another way to relate to math. Units do not have to stay the same within a mathematical activity. Materialized actions as a theoretical idea can guide the design of mathematics learning that is embracing (rather than reducing) complex concreteness as part of learning. This holds the promise to engage people with diverse interests in mathematics learning and unsettle what has previously been conceptualized as a canonical source of mathematics activity.

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